

AD-A165 330

IONIZATION INDUCED INSTABILITY IN AN ELECTRON

1/1

COLLECTING SHEATH(U) S-CUBED LA JOLLA CA

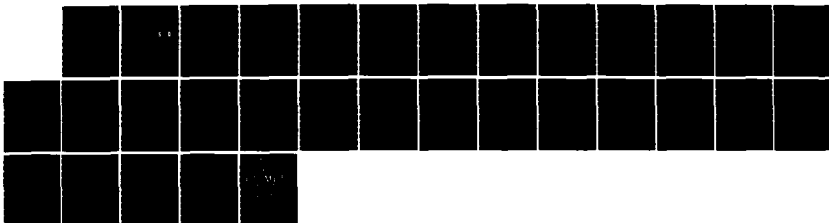
D L COOKE ET AL. JUN 85 SSS-R-85-7343 AFGL-TR-85-0256

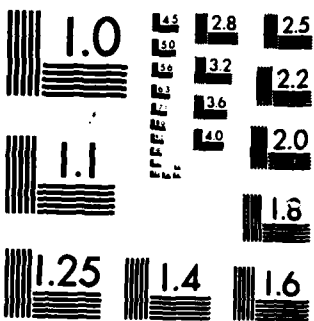
UNCLASSIFIED

F19628-82-C-0081

F/G 9/1

NL





MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

12

AFGL-TR-85-0256

IONIZATION INDUCED INSTABILITY IN AN  
ELECTRON COLLECTING SHEATH

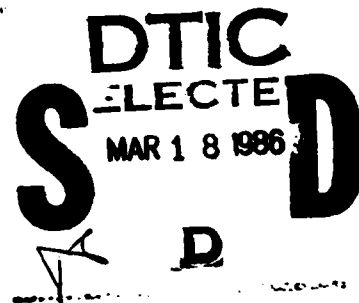
AD-A165 330

D. L. Cooke and I. Katz

S-CUBED

P.O.Box 1620

La Jolla, CA 92038



June 1985

Scientific Report No. 2

Approved for public release; distribution unlimited

DTIC FILE COPY

Air Force Geophysics Laboratory

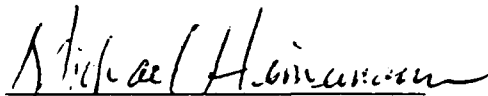
Air Force Systems Command

United States Air Force

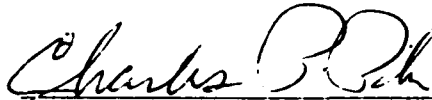
Hanscom Air Force Base, MA 01731

80 3 17 103

"This technical report has been reviewed and is approved for publication"




MICHAEL HEINEMANN  
Contract Manager  
Spacecraft Interactions Branch  
Space Physics Division



CHARLES P. PIKE, Chief  
Spacecraft Interactions Branch  
Space Physics Division

FOR THE COMMANDER

  
RITA C. SAGALYN, Director  
Space Physics Division

This report has been reviewed by the ESD Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

Qualified requestors may obtain additional copies from the Defense Technical Information Center. All others should apply to the National Technical Information Service.

If your address has changed, or if you wish to be removed from the mailing list, or if the addressee is no longer employed by your organization, please notify AFGL/DAA, Hanscom AFB, MA 01731. This will assist us in maintaining a current mailing list.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AD-A165330

## REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>			1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY <b>N/A</b>			3. DISTRIBUTION/AVAILABILITY OF REPORT  Approved for public release; Distribution unlimited	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE <b>N/A</b>				
4. PERFORMING ORGANIZATION REPORT NUMBER(S) <b>SSS-R-85-7343</b>			5. MONITORING ORGANIZATION REPORT NUMBER(S) <b>AFGL-TR-85-0256</b>	
6a. NAME OF PERFORMING ORGANIZATION <b>S-CUBED</b>		6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION <b>DCASMA, San Diego</b>	
6c. ADDRESS (City, State and ZIP Code) <b>P.O. Box 1620 La Jolla, CA 92038</b>			7b. ADDRESS (City, State and ZIP Code) <b>4297 Pacific Highway San Diego, CA 92110</b>	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION <b>Air Force Geophysics Laboratory</b>		8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER <b>Contract F19628-82-C-0081</b>	
8c. ADDRESS (City, State and ZIP Code) <b>Hanscom Air Force Base MA 01731</b>			10. SOURCE OF FUNDING NOS.	
			PROGRAM ELEMENT NO. <b>62101F</b>	PROJECT NO. <b>7661</b>
			TASK NO. <b>11</b>	WORK UNIT NO. <b>AA</b>
11. TITLE (Include Security Classification) <b>IONIZATION INDUCED IN STABILITY IN AN ELECTRON COLLECTING SHEATH</b>				
12. PERSONAL AUTHOR(S) <b>Cooke, D. L. and Katz, I.</b>				
13a. TYPE OF REPORT <b>Scientific Report #2</b>		13b. TIME COVERED FROM _____ TO _____		14. DATE OF REPORT (Yr. Mo., Day) <b>June 1985</b>
15. PAGE COUNT <b>30</b>				
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB. GR.	Spacecraft Charging; Auroral Ionosphere; POLAR Computer Code; Electron Collecting Sheaths; Plasma Electron Sheaths; Probe Theory; Sheath Ion.	
19. ABSTRACT (Continue on reverse if necessary and identify by block number) A numerical investigation of the electron collecting sheath about a positively biased probe in the ionosphere has been conducted to determine the effect of ionization within the sheath. This report is directed at the space charge limited regime appropriate to objects with characteristic size much larger than the ambient Debye length, and uniform surface potential much greater than the ambient plasma temperature. A fluid approximation is employed to model the dynamics of the secondary plasma produced within the sheath. The results indicate that when the secondary ion production exceeds a critical level, the sheath edge propagates outward in an explosive fashion, in a manner similar to the propagation of double layers. This critical level is shown to be equivalent to a mean free path for ionization by electrons $\lambda_{ie} < \sqrt{m_i/m_e} \cdot (D_s/\alpha)$ where $D_s$ is the sheath thickness and $\alpha$ is a geometry dependent parameter. Usual ionosphere-probe interactions appear to be sub-critical, but an enhanced neutral background as might be produced by shuttle contaminants could reach the critical level and trigger the instability. <i>Page 10</i>				
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <b>UNCLASSIFIED/UNLIMITED</b> <input type="checkbox"/> SAME AS RPT <input checked="" type="checkbox"/> OTIC USERS <input type="checkbox"/>			21. ABSTRACT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>	
22a. NAME OF RESPONSIBLE INDIVIDUAL <b>Michael Heinemann</b>			22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL <b>AFGL/PHK</b>

DD FORM 1473, 83 APR

EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

## INTRODUCTION

It is generally accepted that some mechanism for ionizing the neutral gas near an ionospheric probe at high positive potential is required to account for observed levels of electron current collection (Winkler, 1980). Early theoretical studies (Beard and Johnson, 1961; Parker and Murphy, 1967; Linson, 1969) which considered only the collection of ambient electrons tended to underestimate the return currents to an electron emitting vehicle or conversely, overestimate the charge up potential. The predictions of these studies varied widely depending primarily upon the inclusion of magnetic field effects, with Beard and Johnson ignoring the geomagnetic field; Parker and Murphy including it, but mostly ignoring space charge effects, and Linson proposing "turbulence" to transport electrons across the magnetic field. The ionization of neutrals has been considered by Zhulin et al. (1976) and Galeev et al. (1976) but the only attempts at simultaneously including ionization and space charge (but not magnetic fields) are the related studies of Leadon et al. (1980) and Lai et al. (1983).

The present study is based on the configuration shown in Figure 1, where the electrons attracted by the positive probe fall through the sheath region and impact ionized neutral atoms with a cross-section  $\sigma$ . The newly liberated secondary ions move out through the sheath and the secondary electrons are collected by the probe. The secondary charge densities should be self-consistent with the solution of Poisson's equation. Lai et al. predict a non-monotonic current-voltage characteristic that seems to be capable of explaining many observations, but the inclusion of magnetic field effects may ultimately be required.



MIS	CRA&I	<input checked="" type="checkbox"/>
ENC	TAB	<input type="checkbox"/>
Unannounced		<input type="checkbox"/>
Justification		
By		
Distribution		
Priority Codes		
Dist	Review and/or Special	
A-1		

The present study was prompted by a numerical catastrophe that occurs in the Lai model whenever the sheath potential profile becomes non-monotonic. This problem stems from the use of analytic integral expressions for the secondary space charge. In our model, the secondary ion space charge is treated by a fluid model that overcomes this problem. With our model, we have been able to model sheath behavior as the secondary ion density in the (otherwise electron) sheath reaches and exceeds a critical level. Specifically, the critical effect is that with sufficient internal ion space charge, the sheath edge moves outwards to include more negative electron space charge, but in doing so, the interior ion production is proportionally increased leading to continued sheath expansion, or "explosion". This effect is similar to the propagation of double layers (Carlqvist, 1972).

## THE SPACE CHARGE LIMITED SHEATH

This study of sheath ionization is focused on the regime of space charge limited current collection. This is equivalent to the short Debye length limit  $\lambda_D \ll R_0$  where  $\lambda_D$  is the Debye length and  $R_0$  is the probe radius. We also presume that the probe voltage  $V_p \gg kT/e$ . Under these conditions a plasma sheath may be described as composed of three regions: the sheath, the sheath edge, and the pre-sheath. The sheath is the region adjacent to the probe where only the attracted specie is present from the ambient plasma, and the potential decreases with radius,  $r$ , faster than  $r^{-2}$ . This, combined with the assumption of high probe potential, produces a strong electric field that overcomes any orbital motion, and results in near radial trajectories inside of some absorption radius. The sheath edge is the nominal absorption radius for the bulk of the attracted distribution, and is generally a few  $\lambda_D$  in thickness. The presheath is the quasineutral region that connects the sheath to the undisturbed plasma (Parrot et al., 1982).

It is within this picture of the sheath that we propose to examine sheath structure using space charge limited diode models (Chen, 1965).



## SATURATED DOUBLE DIODES

The objectives of this study were two-fold: one, determine the relationship between the probe, plasma, and neutral background that give rise to an explosive condition; and two, model the evolution of the sheath when parameters have exceeded the threshold for sheath explosion. The emphasis is on the first objective. Thus we hypothesize that sheath ionization is simulated by planar and spherical double diodes, then verify and quantify this assumption with our numerical model. By double diode we mean a two electrode device where charge carriers of opposite sign are emitted from the opposing electrodes, and transported across the gap between the electrodes.

The one double diode model to which solutions can be found in the literature is due to Langmuir (1929), and generalized to double layers by Block (1972). This model is illustrated in Figure 2 as a double layer with anode and cathode locations indicated to make the connection with Langmuir's diode. We assume a strong layer where the initial or thermal velocities are negligible compared to potential drop. If the anode does not emit, then we have the usual planar Child-Langmuir diode (Child, 1911; Langmuir, 1913) where the separation  $D_{CL}$ , potential drop  $V$ , and current density  $J$  are related by the expression

$$J_e = \frac{4\epsilon_0}{9} \left( \frac{2e}{m_e} \right)^{1/2} \frac{V^{3/2}}{D_{CL}^2} \quad (2)$$

or, if we use for  $J_e$ , the plasma thermal current,

$$J_e = N_e (kT/2\pi m_e)$$

we may write

$$D_{CL} = 1.26 \lambda_D (eV/kT)^{3/4} \quad (3)$$

where  $\lambda_D = \sqrt{\epsilon_0 kT/N_e^2}$  is the plasma Debye length. If we allow a space charge limited ion current to flow from the anode, we have a symmetric potential profile, and for the same electron current, the separation distance increases to

$$D_d = 1.36 D_{CL},$$

or if the diode gap were fixed, the electron and ion currents would be 1.86 times the limit (2). When both specie currents are space charge limited we see from (2) (regardless of the value of D) that

$$J_e = \sqrt{m_i/m_e} J_i \quad (4)$$

This relation, known as the "Langmuir condition", is the basic stability requirement for a strong double layer. If either flux should deviate, one sign of space charge in the layer will dominate, and the layer will move upstream of the weaker flux so as to satisfy this condition in the moving frame of reference.

This is all well and fine, but in our picture of sheath ionization, the ions are produced throughout the sheath region and rather than the probe surface. To our knowledge the literature does not contain a complete solution for such a diode, although Langmuir (1929) did analyze the problem for single ion release.

The situation for a diode formed of concentric spherical electrodes is even more barren. We do have available the work of Langmuir and Blodgett (1924) who studied the single emitting electrode spherical diode. Their results are primarily tabular and are not reproduced here, but we note that Parker (1980) provides a useful fit to those results. An interesting feature of their results is that for a given potential drop and sphere radii, the limiting current (electrons) is higher for a central cathode than for a central anode. This is not surprising since mutually repulsive particles are happier diverging than

converging. This does lead us to suspect that the double spherical diode will not have the symmetry that allowed us to jump from Eq. (2) to Eq. (4), and that a flux of ions flowing from an inner anode will exceed that suggested by (4) by an amount that we call the anode factor  $\alpha$ , thus

$$F_i = \alpha \sqrt{m_e/m_i} F_e ,$$

where  $\alpha$  will be determined numerically.

## EXPLOSIVE SHEATH IONIZATION

The concept of explosive sheath ionization can be illustrated quite simply. Consider a spherical probe of radius  $r_0$ , with a sheath of volume  $\Lambda$ , area  $A_s$ , and an average ion production rate

$$\bar{S} = \frac{\int_{r_0}^{r_t} \sigma(\epsilon(r)) N_n F_e(r) \cdot 4\pi r^2 dr}{\Lambda}$$

where  $r_t$  is the radius at which electrons have attained sufficient energy,  $\epsilon$ , for the ionization of neutrals with density  $N_n$ . Assuming that orbital motion within the sheath is negligible, we may write

$$F_e(r) \approx F_{es} \left( \frac{r_s}{r} \right)^2$$

where  $F_{es}$  is the presheath enhanced (Parrot, 1980) ambient electron flux to the sheath edge. By also assuming that  $\sigma(\epsilon)$  is constant with  $\epsilon$ , we have

$$\bar{S}(r_t) = 4\pi r_s^2 \Lambda^{-1} \sigma N_n F_{es} (r_t - r_0).$$

The ion flux out through the sheath will be approximately

$$F_i \approx \bar{S}(r_t) \frac{\Lambda}{A_s} \approx \sigma N_n F_{es} (r_t - r_0) \quad (5)$$

We claim that if

$$F_i > \alpha \sqrt{m_e/m_i} F_{es} \quad (6)$$

then the sheath will move outwards,  $\dot{r}_s > 0$ ,  $\dot{r}_t > 0$ , leading directly to  $\dot{r}_i > 0$ , resulting in continued instability of the sheath. From Eq. (5) we can see that condition 6 is equivalent to

$$\lambda_{ie} < \sqrt{\frac{m_i}{m_e}} \frac{D_S}{a}$$

where  $\lambda_{ie} = (\sigma N_n)^{-1}$  is the mean free path of electrons for ionization of neutrals and  $D_S \approx (r_t - r_o)$  is the sheath thickness. A careful estimate of  $\dot{r}_s$  is difficult to obtain since the ion distribution function will depend on the specific potential distribution within the sheath, and the energy dependence of the ionization cross-section. However, we may typify the expansion by noting that it will be limited by the speed of fastest ions, so

$$\dot{r}_s \lesssim \sqrt{\frac{2e \Delta V}{m}} \approx 10^4 \text{ m/s}$$

for oxygen atoms and a probe potential of 50 volts.

There should certainly be mechanisms that will modify and limit the simple analysis present here. Some of these might be:

A complex sheath potential profile may modify ion velocity distributions and cause some to impact the probe. Much of this effect will be included in the following numerical model.

Anomalous resistivity, which if we do not suggest it, someone else will, and could retard electrons in the sheath and increase their space charge.

Trapped orbits, which might be quickly filled by scattered electrons. These will not be truly stable orbits unless there exists

regions where the potential falls off slower than  $r^{-2}$  (Laframboise, 1966) but our numerical studies indicate that as a sheath becomes explosive such regions may indeed appear, even though the problem was initially posed in the strong space charge, short Debye length limit.

Neutral depletion should have a minimal effect. The depletion rate is  $\dot{N}_d \approx \bar{v} \approx \bar{\sigma} F_e N_s = (10^{-6} \leftrightarrow 10^{-3} \text{ sec}^{-1}) N_s$  for  $\bar{\sigma} = 2 \times 10^{-20} \text{ m}^2$ , and a range of possible ionospheric electron fluxes, and where  $N_s$  is the neutral density in the sheath. The repletion rate should be approximately  $\dot{N}_r \approx V_{th} N_n / r_s \approx (10^3 \leftrightarrow 10^5 \text{ sec}^{-1}) N_n$  for 1-10 m sheaths, and 100-1000 °K neutral temperature range, so we see that  $N_s = N_n$ .

Probe impedance will have a significant effect and is probably the major distinction between this study and that of Lai et al. They modeled their probe as part of constant current circuit where the probe voltage would fall as a result of the net increase in electron current that a sheath explosion would produce. Our probe is a constant voltage, zero impedance device which can support arbitrary large currents, which is a bit removed from the case of an electron emitting rocket, but does relate to a small probe on a large vehicle biased by a hefty power supply.

From an electric circuit viewpoint, this sheath explosion would appear as a collapse of the sheath impedance.

## THE NUMERICAL MODEL

A one-dimensional fluid Poisson code, FLOMO, was written to model the sheath ionization problem. FLOMO can model either a single planar coordinate or a spherical radial coordinate. A number of gridding schemes were considered, but uniform  $x$  (planar) and  $r$  (spherical) spacings proved to be the most practical. The Poisson-solver is a simple tri-diagonal method (Richtmyer and Morton, 1967) that incorporates the following charge densities:

The repelled ambient ion density was modeled everywhere as  $N_{ai} = N_0 \exp(-e V/kT)$ . For the calculation of the attracted ambient electron density, a sharp edge sheath approximation was used to separate different approximations that are matched at the sheath edge. For spherical geometry the sheath edge conditions were taken from Parrot et al. (1982) who found that for  $V_p \gg kT/e$ , the sheath edge had a potential  $V_s = 0.49 kT/e$ , number density  $N_{es} = 0.611 N_0$ , and flux density  $F_{es} = 1.45 F_{e0}$ . Outside of the sheath the attracted electron number density was taken from a geometric fit to the results of Parrot et al. We start with

$$N_g(r) = N_0 \left( 1 - \frac{\Omega(r)}{4\pi} \right)$$

as the geometric density that would result from shadowing by a sheath subtending a solid angle

$$\Omega(r) = 2\pi \left( 1 - \sqrt{1 - (R_s/r)^2} \right)$$

At the sheath edge  $N_{gs} = 0.5 N_0$ . A good fit to the results of Parrot et al. is given by

$$N_e(r) = N_g(r) + (0.611 \cdot N_o - N_{gs}) \cdot \left(\frac{\rho}{2\pi}\right)^2$$

Inside the sheath edge, orbital motion is ignored and the attracted density is calculated by the usual continuity argument

$$N_e(r) = \left(\frac{r_s}{r}\right)^2 F_{es}/u(r) \quad (7)$$

where velocity  $u(r)$  is given by energy conservation

$$u(r) = \sqrt{u(r_s)^2 + 2e(V(r) - V_s)/m} \quad (8)$$

and where

$$u(r_s) = F_{es}/N_{es}$$

The secondary ion space charge is modeled as a fluid, using the continuity equation

$$\frac{\partial N}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 (Nu) = S(r) \quad (9)$$

and the momentum equation

$$\frac{\partial (Nu)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 (Nuu) = NF(r) \quad (10)$$

where  $F = qE/m$  is the single particle acceleration and  $E = -\nabla V$ . The



closure of Eqs. (9) and (10) is provided by Poisson's equation. These equations are solved by finite differences using full upwind and backwards time centering (explicit) on the convective terms (Richtmeyer and Morton, 1967). This approach was chosen because of its robustness and its ability to handle non-monotonic potential distributions, which is the particular advantage over an integral kinetic method. More rigorous methods such as a turning point formalism (Laframboise, 1966) or a fully differenced Vlasov approach (Rotenberg, 1983) was beyond the scope of this work. We have performed an analysis to determine the extent of error introduced by our fluid model.

Consider a radial probe of negligible radius and a potential distribution that is linearly decreasing with  $r$ , such that the force is constant with  $r$ . If we add to this a constant source rate, then Eqs. (9) and (10) may be solved to give

$$N = \sqrt{\frac{7}{18}} \cdot S \cdot \sqrt{\frac{r}{F}} = 0.62 S \sqrt{\frac{r}{F}}$$

and

$$u = \sqrt{\frac{2F}{7}} \cdot \sqrt{Fr} = 0.53 \sqrt{Fr}$$

Kinetically we have

$$N(r) = \int_0^r f(r, u) du = \int_0^r S \cdot \left( \frac{r'}{r} \right)^2 \cdot \frac{dr'}{u(r, r')}$$

where  $u(r, r') = \sqrt{2F(r-r')}$ . This integrates to

$$N_k = 0.75 S \sqrt{\frac{r}{F}}$$

and similarly we compute

$$\bar{u} = \frac{1}{N} \int_0^r u(r) f(r,u) du = 0.44 \sqrt{Fr}$$

We conclude from this simple analysis that the fluid approach typically introduces an acceptable error, and we have the reassuring result that

$$Nu = N_k \bar{u}.$$

The secondary electron space charge is ignored unless local trapping occurs. We justify this by observing that in the absence of trapping, the electron space charge is less than the ion space charge by a factor of  $\sqrt{m_e/m_i}$ . Throughout the iterative solution of our equations we inspect the potential distribution for the appearance of an electron trap, or well. Let us assume that a well of depth  $H$  has appeared, and that in the well the electron distribution is Maxwellian for all trapped electron energies, zero for untrapped energies, and normalized to the central ion density,  $N_{si}(r_w)$ , thus

$$\begin{aligned} N_{se}(r) &= N_{si}(r_w) \cdot \left( \frac{m_e}{2\pi kT} \right)^{3/2} \cdot \exp(e(V(r) - V(r_w))/kT) \\ &\cdot 4\pi \int_0^{u(H)} e^{-mu^2/2kT} u^2 du \\ &= N_{si}(r_w) \cdot \exp((V(r) - V(r_w))/kT) \\ &\cdot \left[ \operatorname{erf} \sqrt{\frac{H}{kT}} - 2 \sqrt{\frac{H}{\pi kT}} \exp\left(-\frac{H}{kT}\right) \right] \end{aligned}$$

A secondary electron temperature of 1 eV was assumed in the calculations. These various charge densities are summed and possibly adjusted according to a charge stabilization algorithm (Cooke, 1983) that guarantees stability in the Poisson solution without iterate mixing (Parker and Sullivan, 1969, 1970). Briefly, this algorithm derives from a plausibility argument which recognizes that in the discretization of Poisson's equation, space charge features (such as the sheath edge) that are nonlinear with position and potential, and numerical errors may both become artificially amplified when summed over discrete elements. The charge stabilization technique inspects for excessive total discrete charge and reduces it to a limit that is consistent with numerical stability.

Our iteration scheme was to simply timestep the fluid equations (9) and (10) for generally a few milliseconds, iteratively solve Poisson's equation, and return to the fluid equations.

## RESULTS

The present results consist of six models: A planar configuration with either no local (secondary) ions, an anode ion source, or volume ion production (ionization of neutrals); and a spherical configuration with the same local ion options. The planar model with anode ions and no local ions serves as a check on our numerics since those results may be compared with the analytic theory of Langmuir (1929). These models generally employed 100 grid points to span 30 meters (planar) or 12 meters (spherical). All six models employed the same ambient plasma parameters which are summarized in Table 1. The steady state results are presented in Table 2, where the sheath edge,  $D_S$ , is found from the solution as the  $-0.5$  kT/e point. The results for both the anode plasma and ionization models represent the saturation limit, i.e., if in either case the ion production is increased, the sheath edge begins to move outwards. The same is true for the spherical models. The sheath thickness given for the spherical models is  $D_S = R_S - R_0$ . The theoretical prediction for  $D_S$  comes from Langmuir and Blodgett (1924) with the presheath current enhancement of Parrot et al. (1982).

The explosive character of the sheath was investigated for the four cases that involved saturation limits, but we present here a study of only the spherical model with critical ionization. Figure 3 is a voltage versus radius profile for the 2 meter radius spherical probe, with a neutral density of  $5.0 \times 10^{16}$ . Notice that the sheath radius is  $\sim 6.4$  meters which is essentially the results for no ionization. The charge densities are presented in Figure 4, where the attracted electron density may be seen to dominate everywhere, except just outside the sheath. This artifact is primarily a numerical error relating to the precision with which the sheath edge may be located for the fit to the Parrot et al. results. This may also be observed in the filtered total charge density (labeled as QUSD for "Q that was used") shown in Figure 5. The QUSD values are normalized to the ambient plasma density. From a comparison of Figures 5 and 6, one can see how the positive overshoot error is limited by the charge stabilization process. The recognition

of this overshoot as an error is accomplished by comparison with the potential, and the enforcement of a charge limit that is linear with potential.

In Figure 6, the neutral density has been increased to  $N_n = 7.0 \times 10^{16}$ . The increase is modest, but the effect is dramatic resulting in an increase in  $R_s$  from 6.4 to 7.7 meters, with a 45 percent increase in sheath area and ambient electron current. Figure 7 shows the charge density profiles where we see that electrons are still dominant inside the sheath.

If the neutral density is incremented to  $8.0 \times 10^{16} \text{ m}^{-3}$ , the sheath explosion begins. Figure 8 is a voltage profile snapshot, at 1.3 msec, and Figure 9 is the charge profile. Notice how the ion density begins to dominate in some regions. As this occurs, the electric field that removes the ions is weakened, ion removal is reduced, and the density increases. The calculation was halted at 2.2 msec when the sheath hit the outer boundary at 14 meters. This final set of profiles are shown in Figures 10, 11 and 12.

## CONCLUSIONS

This study of sheath ionization has been composed of many approximations which would allow one to question the accuracy of the results. However, our ability to match with other methods suggest that the accuracy is adequate. Furthermore, while the determination of the critical parameters is subject to our numerical accuracy, the basic instability is not, and we are led to the conclusion that with sufficiently low probe impedance, and sufficiently high neutral density, an electron collecting sheath will become explosive. We may also conclude that for a neutral density that is a factor of 2 or so below the critical level, ionization has little impact on electron collection.

The determination of the equivalent anode factor for the case of a spherical probe with sheath ionization is subject to our numerical accuracy, and this interesting number deserves closer scrutiny. However, we may still obtain a conservative estimate of the critical neutral density from the planar results since the anode factor is unity (the limit of  $D_s \ll R_0$ ), and the planar sheath is thickest. Thus we return to Eq. (4) and (5), using  $\bar{S} \approx \sigma F_e N_n$  and  $\Lambda/A_s \approx D_d$ , to obtain

$$N_n \leq (\sigma D_d)^{-1} \sqrt{\frac{m_e}{m_i}}$$

or using Eq. (3),

$$N_n \leq (\sigma \lambda_D)^{-1} \left( \frac{kT}{eV} \right)^{3/4} \sqrt{\frac{m_e}{m_i}} \quad (11)$$

For a daytime shuttle environment, we might choose  $kT/e = 0.1$  eV,  $N_e = 1 \times 10^{11} \text{ m}^{-3}$ ,  $\lambda_D = 0.74$  cm, and  $\sigma = 2 \times 10^{-20} \text{ m}^{-3}$ . For these conditions Eq. (11) predicts that a surface at a potential of 1 kilovolt would develop an explosive sheath at  $N_n \geq 2 \times 10^{16} \text{ m}^{-3} \approx 5 \times 10^{-7}$  Torr. In both this case, and the previous model plasma, the critical neutral density is quite high compared to the natural background, but within the possible contamination range for the shuttle.

#### ACKNOWLEDGMENT

The author would like to acknowledge Milt Chapman for his aid in setting up the fluid numerics, and Donald E. Parks for his consulting and advice throughout this project.

Table 1. Model Plasma Parameters

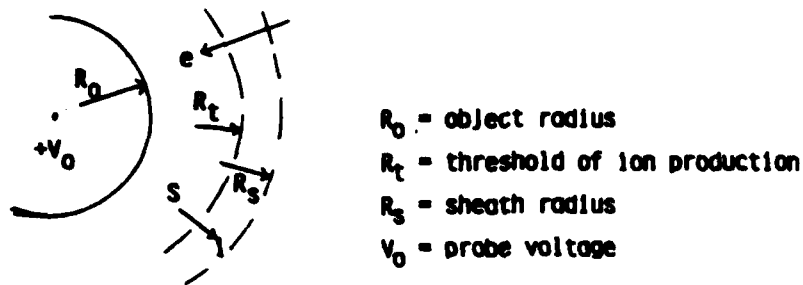
$N_0 = 1.2 \times 10^9 \text{ m}^{-3}$                        $\sqrt{m_i/m_e} = 242.2$  for Oxygen  
 $kT = 0.04 \text{ eV}$                                $\lambda_D = 4.3 \times 10^{-3} \text{ m}$   
 $F_{eo} = 4.0 \times 10^{13} \text{ m}^{-2} \text{ s}^{-1}$   
 For  $V_p = 50 \text{ volts}$   
 $D_{CL} = 11.4 \text{ cm}$                                $D_d = 1.36 \cdot D_{CL} = 15.5 \text{ m}$   
 For  $R_0 = 2.0$  (spherical)  
 $D_S = 4.0 \text{ m}$  (Langmuir, Blodgett, 1924, Parrott et al. 1982)

Table 2.

Model	Sheath Thickness in meters		Anode Factor $\alpha$	Critical Neutral Density $N_n \text{ (m}^{-3}\text{)}$
	Model	Theory		
Planar w/o Local Ions	11.6	11.4	-	
Planar w Anode Plasma	15.9	15.5	1.05	
Planar w Ionization	16.4	-	1.08	$1.4 \times 10^{16}$
Spherical w/o Local Ions	4.4	4.0	-	
Spherical w Anode Plasma	6.5	-	2.1	
Spherical w Ionization	5.7	-	2.3	$7.0 \times 10^{16}$

Sheath thickness, anode factors, and critical neutral density levels for the six models. Table 1 describes the plasma, the anode potential was 50 volts in all cases, and the spherical probe radius was 2 m.





$S$  = ion production rate

$$= \sigma_{ei} \cdot F_e(r) \cdot N_n$$

$\sigma_{ei}$  = ionization cross-section

$N_n$  = neutral density background

$F_e$  = accelerated ambient electron flux

Figure 1. Ionization model configuration.

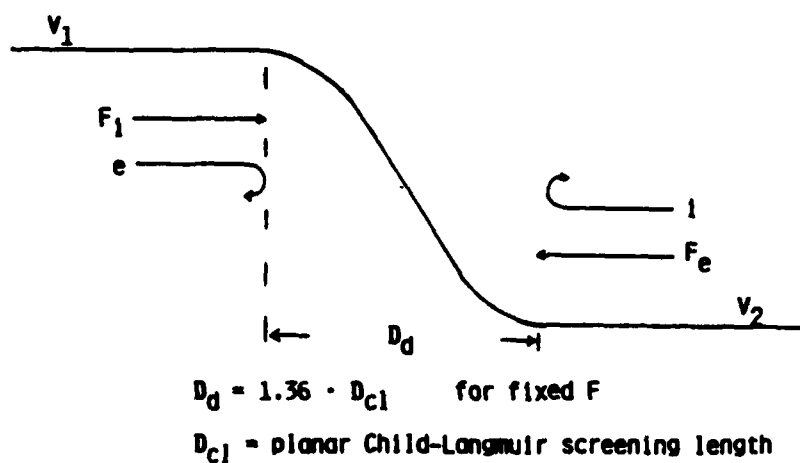


Figure 2. Saturated equilibrium double layer separating two plasmas with a potential difference  $\Delta V = V_1 - V_2$ .

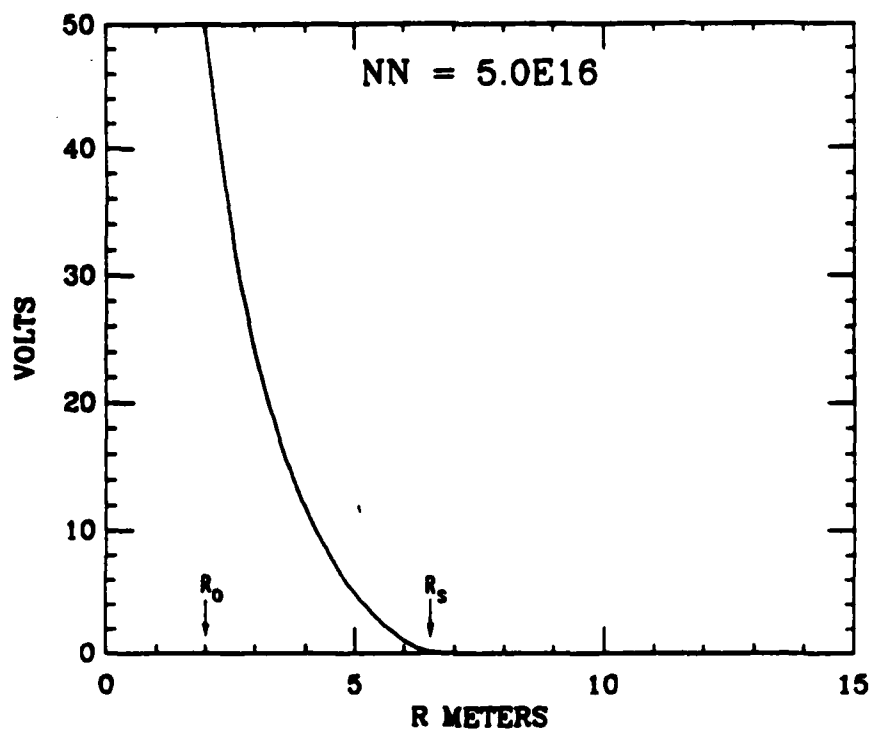


Figure 3. Potential profile of a spherical probe of radius  $R_0 = 2$  m, with the plasma of Table 1, a surface potential of 50 volts, and a neutral density of  $N_n = 5.0 \times 10^{16} \text{ m}^{-3}$ .

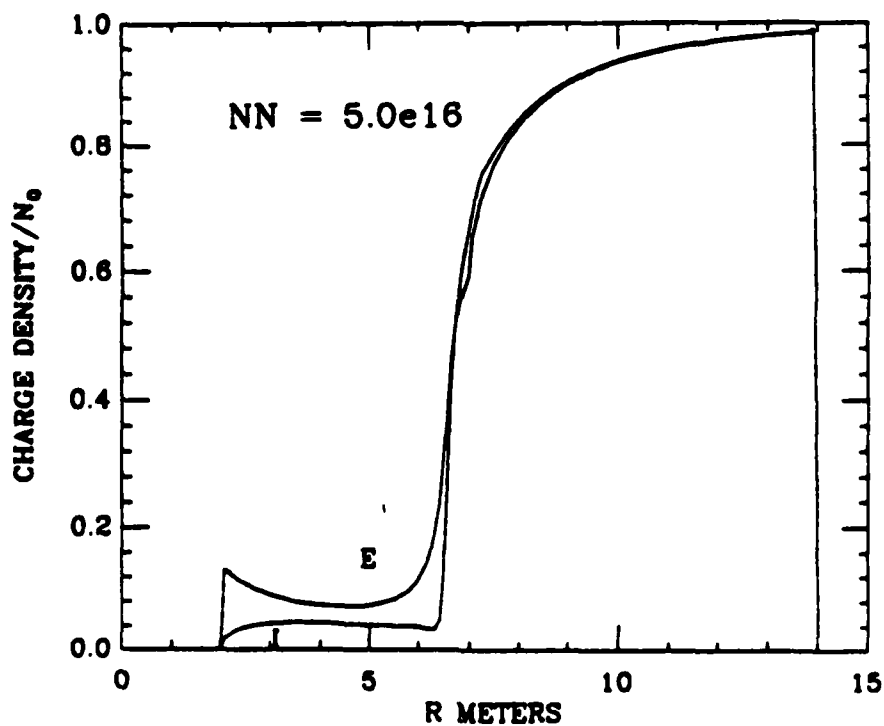


Figure 4. Normalized charge density profile corresponding to Figure 3.

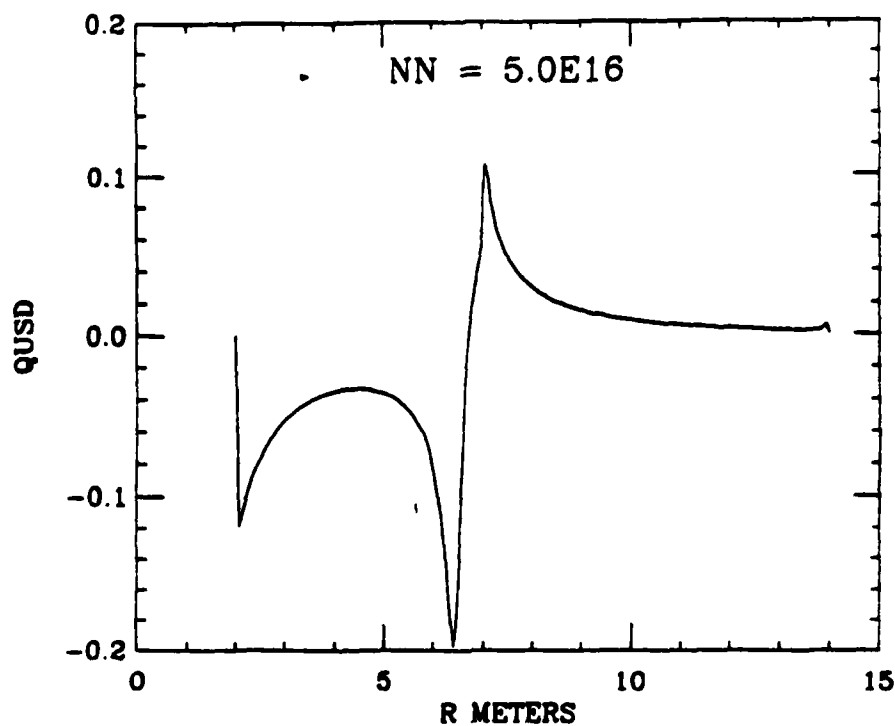


Figure 5. Filtered charge density profile corresponding to Figures 3 and 4.

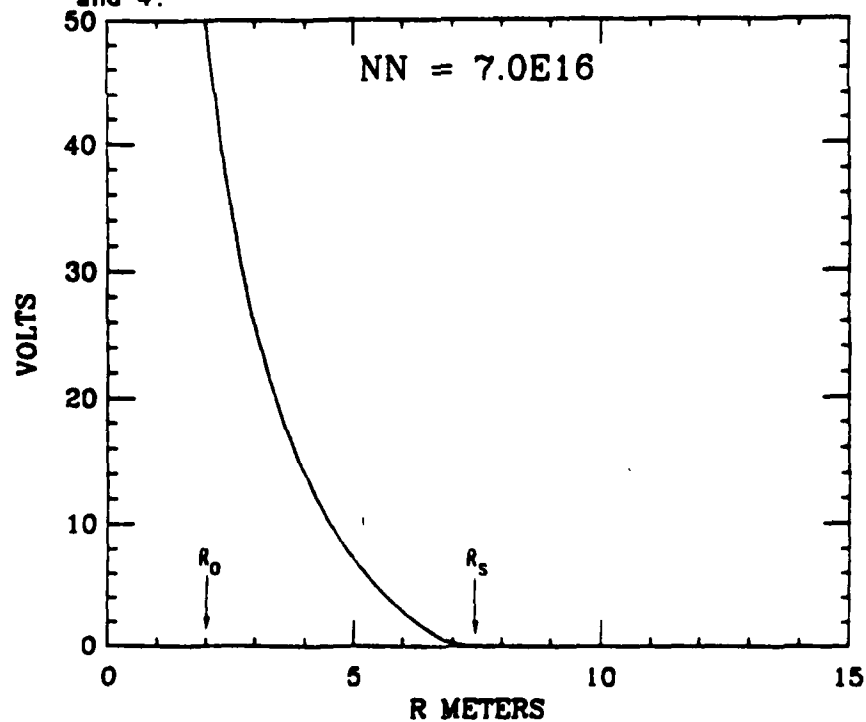


Figure 6. Potential profile for the same spherical probe and plasma as Figure 3, with neutral density increased to  $7.0 \times 10^{16} \text{ m}^{-3}$ , the limiting stable density.

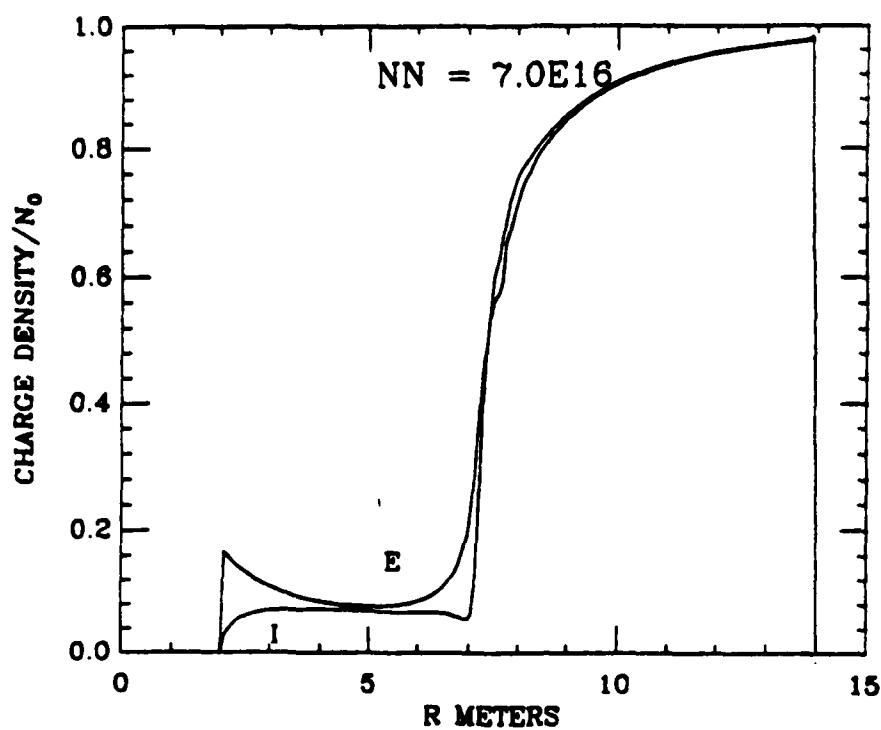


Figure 7. Normalized charge density profile corresponding to Figure 6.

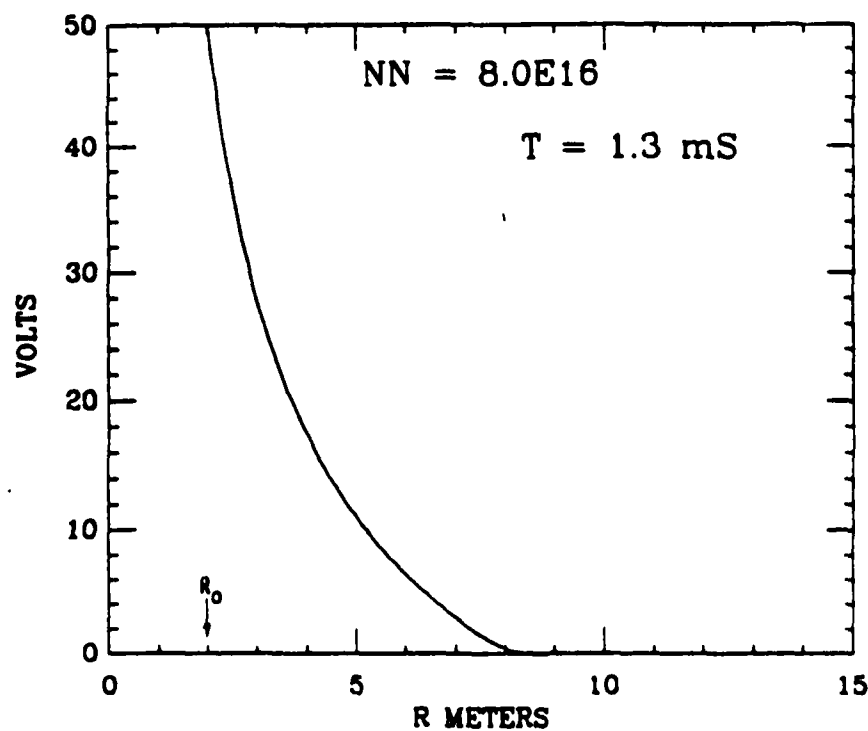


Figure 8. Potential profile of spherical probe-plasma  $1.3 \times 10^{-3}$  seconds after increasing the neutral density to  $8.0 \times 10^{16} \text{ m}^{-3}$ .

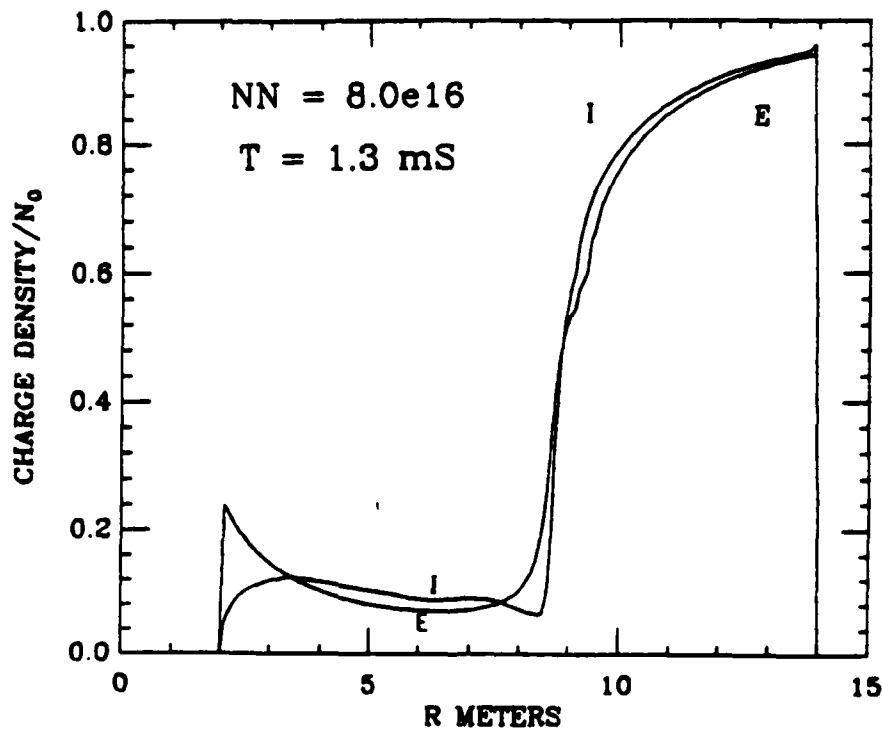


Figure 9. Normalized charge density profiles corresponding to Figure 8. Note that the ion density exceeds that of the electrons near mid-sheath.

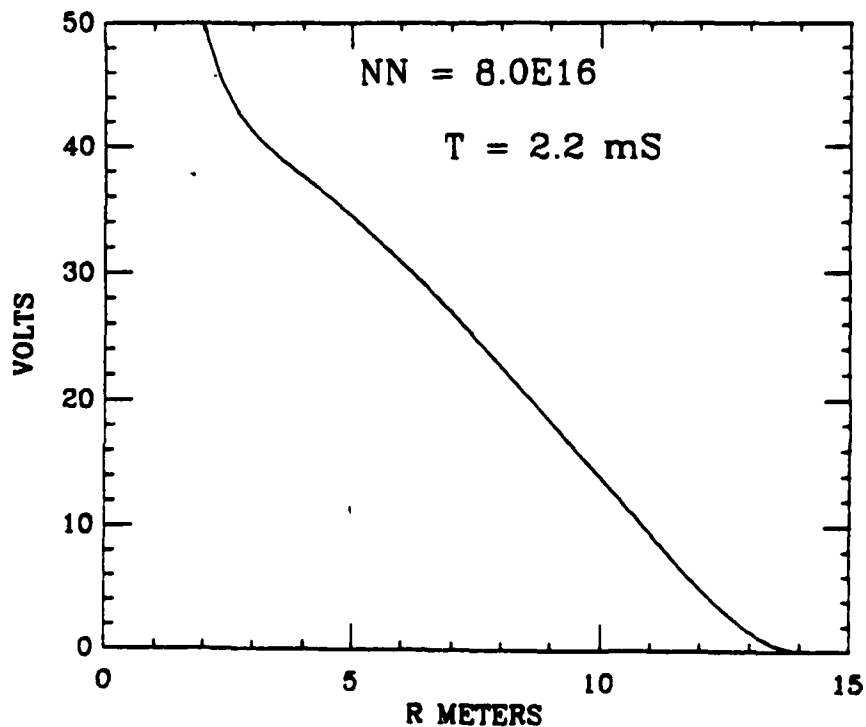


Figure 10. Potential profile at the conclusion of the calculations at 2.2 msec. The sheath has expanded out to the model boundary at 14 meters.

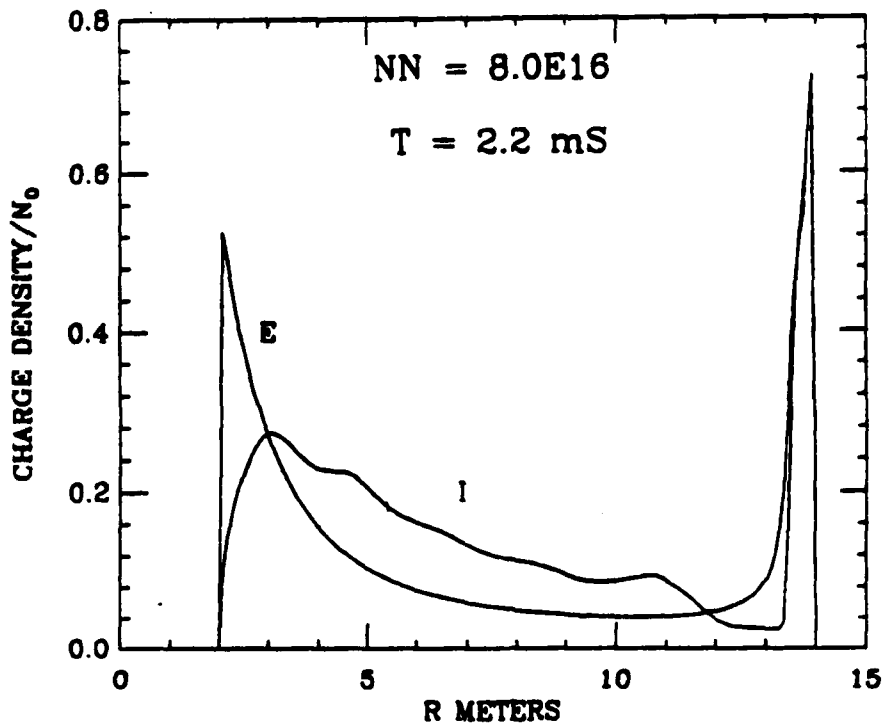


Figure 11. Charge density profile corresponding to Figure 10; ion density dominates most of the sheath.

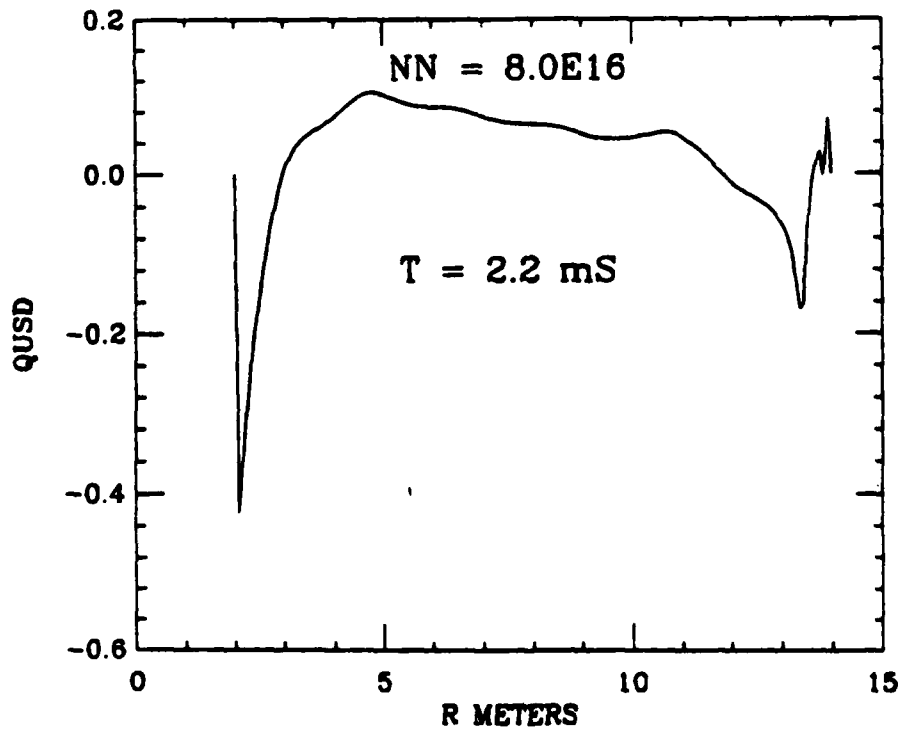


Figure 12. Filtered total charge density corresponding to Figures 10 and 11.

## REFERENCES

- Beard, D. B. and F. S. Johnson, "Ionospheric Limitations on Attainable Satellite Potential," *J. Geophys. Res.*, 66, 1961.
- Block, L. P., "Potential Double Layers in the Ionosphere," *Cosmic Electrodynamics*, 3, 1972.
- Carlqvist, P., "On the Formation of Double Layers in Plasmas," *Cosmic Electrodynamics*, 3, 1972.
- Chen, F. F., "Electric Probes," in *Plasma Diagnostic Techniques*, edited by R. H. Huddleston and S. L. Leonard, Vol. 21 of Pure and Applied Physics, Academic Press, 1965.
- Child, C. D., "Discharge from Hot CaO," *Phys. Rev.*, 32, 492, 1911.
- Cooke, D. L., I. Katz, M. J. Mandell, J. R. Lilley, Jr. and A. G. Rubin, "A Three-Dimensional Calculation of Shuttle Charging in Polar Orbit," *Spacecraft Environmental Interactions Technology-1983*, NASA CP-2359, AFGL-TR-85-0018, 229-234, 1985.
- Galeev, A. A., E. V. Mishin, R. Z. Sagdeev, V. D. Shapiro and V. I. Shevbenko, "Discharge in the Region Around a Rocket Following Injection of Electron Beams into the Ionosphere," (in Russian), *Dokl. Akad. Nauk SSSR*. 231, 71-74, 1976.
- Laframboise, J. G., "Theory of Spherical and Cylindrical Langmuir Probes in a Collisionless, Maxwellian Plasma at Rest," *UTIAS Report No.* 100, 1966.
- Lai, S. T., H. A. Cohen, K. H. Bhavnani and M. Tautz, "Sheath Ionization Model of Beam Emissions from Large Spacecraft," *Spacecraft Environmental Interactions Technology-1983*, NASA CP-2359, AFGL-TR-85-0018, 253-262, 1985.
- Langmuir, I., "The Effect of Space Charge and Residual Gases on Thermionic Currents in High Vacuum," *Phys. Rev.*, 2, 450, 1913.
- Langmuir, I. and K. B. Blodgett, "Currents Limited by Space Charge Between Concentric Spheres," *Phys. Rev.*, 24, 49, 1924.
- Langmuir, I., "The Interaction of Electron and Positive Ion Space Charges in Cathode Sheaths," *Phys. Rev.*, 33, 954, 1929.
- Langmuir, I. and K. B. Blodgett, "Currents Limited by Space Charge Between Concentric Spheres," *Phys. Rev.*, 1, 49, 1924.

- Leadon, R. E., A. J. Woods, E. P. Wenaas and H. H. Klein, "Analytical Investigation of Emitting Probes in an Ionized Plasma," AFGL-TR-81-0138, 1980.
- Parker, L. W., "Plasmasheath-Photosheath Theory for Large High-Voltage Space Structures," in Space Systems and Their Interactions with Earth's Space Environment, Prog. in Astronautics and Aeronautics, Vol. 71, edited by H. B. Garrett and C. P. Pike, 1980.
- Parker, L. W. and B. L. Murphy, "Potential Buildup on an Electron-Emitting Ionospheric Satellite," J. Geophys. Res., 72, N5, 1967.
- Parker, L. W. and E. C. Sullivan, "Numerical Solution of the Poisson-Vlasov System for a Spherical Electrostatic Probe," presented at S.I.A.M. Fall Meeting, 1970.
- Parrot, M.J.M., L.R.O. Storey, L. W. Parker and J. G. Laframboise, "Theory of Cylindrical Langmuir Probes in the Limit of Vanishing Debye Number," Phys. Fluids, 24(12), 1982.
- Richtmyer, R. D. and K. W. Morton, Difference Methods for Initial Value Problems, Interscience, 1967.
- Rotenberg, M., "Transport Theory for Growing Cell Populations," J. Theoretical Biology, 103, 181-191, 1983.
- Winkler, J. R., "The Application of Artificial Electron Beams to Magnetospheric Research," Rev. Geophys. and Space Phys., 18, 3, 1980.
- Zhulin, I. A., I. M. Kopaev, T. E. Koshelets and A. M. Moskalenko, "On Rocket Charge Neutralization in "Zarnitza" Experiments, Report, Inst. of Terr. Magn.



END  
FILMED

4-86

DTIC